|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | X | X squared | Y | Y squared | X times Y |
|  | 1 | 1 | 1 | 1 | 1 |
|  | 2 | 4 | 2 | 4 | 4 |
|  | 3 | 9 | 3 | 9 | 9 |
|  | 4 | 16 | 4 | 16 | 16 |
|  | 5 | 25 | 5 | 25 | 25 |
| Total | 15 | 55 | 15 | 55 | 55 |
|  | sum X | sum X squared | sum y | sum Y squared | sum X times y |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | X | x | X squared | Y | Y | Y squared | X times Y |
|  | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|  | 2 | 2 | 4 | 3 | 3 | 9 | 6 |
|  | 3 | 3 | 9 | 2 | 2 | 4 | 6 |
|  | 4 | 4 | 16 | 4 | 4 | 16 | 16 |
|  | 5 | 5 | 25 | 5 | 5 | 25 | 25 |
| Total | 15 | 15 | 55 | 15 | 15 | 55 | 54 |
|  | sum X | sum X | sum X squared | sum y | sum y | sum Y squared | sum X times y |

Think about the 2 columns with a background in yellow (top table). Squaring them is like having 2 columns for x and two columns for y (the lower table). The pairs then form the 2 x columns multiplied and the pairs from column y are multiplied. That is, when numbers are squared it is like have two identical columns (lower table). However, when the pairs of numbers in column x are multiplied times the numbers in column y they may or may not be the same. The numbers in row 2 and row 3 are different. Notice that the combined result for those two numbers (6 + 6 = 12) is lower than when they are squared or multiplied (9 + 4 = 13). Consequently, when we compare the sum of squares of x and y (combined) with x times y we know that if they are not perfectly matched and x times y will be smaller. As the pairs become even more different then x times y will be proportionately smaller. As the numbers change the x times y results until the pairs are “most opposite.” That is, when the largest number in one column (x) is paired with the smallest number in the second column (y). Then the next largest number is paired with the next smallest number and so on until the smallest number in the first column is paired with the largest number in the second column. This last operation will result in the x times y summed result being the smallest possible.

HOWEVER

Because we use little x and little y in the formula:



Things change a bit. The largest numbers will be about half the size they were be (little x = X minus the mean of X) and the smallest numbers will be about the same size as the largest except they will be negative. All of the underlying notions mentioned above will remain except when a negative is multiplied times a positive it becomes negative and when two negatives are multiplied the result is positive. I would not get too involved with this latter operation. In order to understand the above formula just stick with the above numbers. Think of the denominator as merely the combination (almost like a mean of the two sums).



The above formula then is a ratio (a comparison) of the highest possible multiplications (the denominator) and the degree of mixed upness of the two sets (sum of little x times little y).